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Kappa-symmetric deformations of M5-brane dynamics

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Abstract: We calculate the first supersymmetric and kappa-symmetric derivative deformation of the M5-brane worldvolume theory in a flat eleven-dimensional background. By applying cohomological techniques we obtain a deformation of the standard constraint of the superembedding formalism. The first possible deformation of the constraint and hence the equations of motion arises at cubic order in fields and fourth order in a fundamental length scale l. The deformation is unique up to this order. In particular this rules out any induced Einstein-Hilbert terms on the worldvolume. We explicitly calculate corrections to the equations of motion for the tensor gauge supermultiplet.

Keywords: M-Theory, Superspaces, Supersymmetric Effective Theories.

Contents

1. Introduction

In the web of dualities relating ten-dimensional string theories and eleven-dimensional supergravity, the M5-brane plays a central role. It is important therefore to understand the structure of the effective field theory which describes M5-brane dynamics. The multiplet which describes the linearised worldvolume dynamics of an M5-brane in eleven dimensions is the on-shell $N = (2, 0)$, $D = 6$ tensor multiplet [1] whose bosonic sector contains a 2-form gauge potential with self-dual 3-form field strength as well as five scalars. The full non-linear equations of motion for the worldvolume theory of the M5-brane in a general eleven-dimensional supergravity background were first constructed using the superembedding formalism [2, 3]. The non-linear theory constructed in [2, 3] is the tensor gauge theory analogue of supersymmetric Dirac-Born-Infeld theory which is the non-linear theory describing the worldvolume dynamics of the D-branes of type-II string theory [4 − 6]. Since one can obtain the IIA theory by reduction of the eleven-dimensional theory on a circle, one can relate the worldvolume dynamics of the M5-brane to the dynamics of the D4-brane by double dimensional reduction and indeed it was shown in [3] that the non-linear tensor gauge theory correctly reproduces the Dirac-Born-Infeld equations of motion after such a reduction. A covariant action which reproduces the equations of motion was subsequently constructed in [7].

Just as the Dirac-Born-Infeld theory describing the worldvolume dynamics of D-branes receives derivative corrections, one expects the tensor gauge theory that describes the fluctuations of the M5-brane to receive derivative corrections. These corrections will involve

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the derivative of the self-dual 3-form field strength h_{abc}, induced curvature correction terms and higher derivative terms for the fermion fields. We will show how the form of these derivative corrections can be systematically constrained by supersymmetry and kappa-symmetry. Our approach is directly analogous to that of [8, 9] where it was shown that the deformations of the standard superembedding constraints can be classified according to spinorial cohomology [10, 11]. We will show that the first possible deformations to the equations of motion are cubic in the fields and appear at fourth order in a fundamental length parameter l. In particular this implies that there is no induced Einstein-Hilbert type term in the effective action.

2. The M5-brane as a superembedding

We now briefly describe the superembedding formalism [12] as applied to the linearised description of the M5-brane [13]. We consider the embedding of an $N = (2, 0)$, $D = 6$ superspace, $\mathcal{M}$, into a $N = 1$, $D = 11$ superspace, $\underline{\mathcal{M}}$. The coordinates of $\mathcal{M}$ ($\underline{\mathcal{M}}$) are denoted by $z^M = (x^m, \theta^\mu)$ ($z^{\underline{M}} = (x^{\underline{m}}, \theta^{\underline{\mu}})$), with Latin indices for bosonic coordinates and Greek for fermionic. Cotangent frames are related by the vielbein matrices, $E^A = (E^a, E^\alpha) = dz^M E_M{}^A$ ($E^{\underline{A}} = (E^{\underline{a}}, E^{\underline{\alpha}}) = dz^{\underline{M}} E_{\underline{M}}{}^{\underline{A}}$). Here the Latin indices a ($\underline{a}$) are vector indices of the group Spin$(1, 5)$ (Spin$(1, 10)$). The Greek indices $\underline{\alpha}$ are spinor indices of Spin$(1, 10)$ while the indices α are multi-indices containing a Spin$(1, 5)$ spinor index and a Spin$(5) \cong Usp(4)$ spinor index. The embedding splits target space indices into tangent and normal indices on the worldvolume, $\underline{A} = (A, A')$. The normal indices a' are Spin(5) vector indices while α' are again multi-indices containing Spin$(1, 5)$ and Spin(5) spinor indices. Later we will introduce the 'two-step' notation and explicitly replace a subscript α with a subscript pair αi. A subscript α' is replaced by a pair ${}^{\alpha}_{i}$. Lower and upper α indices distinguish left and right handed Weyl spinors in six dimensions.

The embedding is a map, $f : \mathcal{M} \longrightarrow \underline{\mathcal{M}}$ and it induces the pullback map relating the two frames,

$$f^* E^{\underline{A}} = E^A E_A{}^{\underline{A}}, \qquad E_A{}^{\underline{A}} = E_A{}^M \partial_M z^{\underline{M}} E_{\underline{M}}{}^{\underline{A}}. \tag{2.1}$$

The standard embedding condition $E_\alpha{}^{\underline{a}} = 0$ was shown in [13] to fix the worldvolume multiplet to be the $N = (2, 0)$, $D = 6$ supersymmetric tensor gauge theory multiplet and it implies the full non-linear equations of motion [2, 3] which describe the fluctuations of the M5-brane in a general eleven-dimensional supergravity background. Imposing the embedding condition, one can parametrise the embedding matrix as follows,

$$E_a{}^{\underline{a}} = \left(\begin{array}{c|c} u_a{}^{\underline{a}} & \Lambda_a{}^{\beta'} u_{\beta'}{}^{\underline{\alpha}} \\ \hline 0 & u_\alpha{}^{\underline{\alpha}} + h_\alpha{}^{\beta'} u_{\beta'}{}^{\underline{\alpha}} \end{array} \right). \tag{2.2}$$

Here we have an element of SO$(1, 10)$, written $u_{\underline{b}}{}^{\underline{a}}$, which is split according to $u_{\underline{b}}{}^{\underline{a}} = (u_b{}^{\underline{a}}, u_{b'}{}^{\underline{a}})$. We denote by $u_{\underline{\beta}}{}^{\underline{\alpha}}$ the corresponding element of Spin$(1, 10)$ which obeys $(\Gamma^{\underline{b}})_{\underline{\alpha}\underline{\beta}} u_{\underline{b}}{}^{\underline{a}} = u_{\underline{\alpha}}{}^{\underline{\gamma}} u_{\underline{\beta}}{}^{\underline{\delta}} (\Gamma^{\underline{a}})_{\underline{\gamma}\underline{\delta}}$ and is split according to $u_{\underline{\beta}}{}^{\underline{\alpha}} = (u_\beta{}^{\underline{\alpha}}, u_{\beta'}{}^{\underline{\alpha}})$.

The relevant information can be derived from the torsion equation which follows from applying the pullback map to the definition of the target space torsion,

$$d f^* E^{\underline{A}} = f^* T^{\underline{A}}. \tag{2.3}$$

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We take the target space to be flat $N = 1$ $D = 11$ superspace with the only non-zero components of the target space torsion given by

$$T_{\underline{\alpha}\underline{\beta}}{}^{\underline{c}} = -i(\Gamma^{\underline{c}})_{\underline{\alpha}\underline{\beta}}\,. \tag{2.4}$$

In components the torsion equation reads

$$\nabla_A E_B{}^{\underline{C}} - (-1)^{AB}\nabla_B E_A{}^{\underline{C}} + T_{AB}{}^C E_C{}^{\underline{C}} = (-1)^{A(B+\underline{B})} E_B{}^{\underline{B}} E_A{}^{\underline{A}} T_{\underline{AB}}{}^{\underline{C}}\,. \tag{2.5}$$

In analysing the torsion equation one encounters the Lie algebra-valued quantity $X_{A,\underline{a}}{}^{\underline{b}} = \nabla_A u_{\underline{a}}{}^{\underline{c}} u^{-1}{}_{\underline{c}}{}^{\underline{b}}$. The corresponding quantity with spinorial indices is related by gamma matrices,

$$X_{A,\underline{\alpha}}{}^{\underline{\beta}} = \nabla_A u_{\underline{\alpha}}{}^{\underline{\gamma}} u^{-1}{}_{\underline{\gamma}}{}^{\underline{\beta}} = \frac{1}{4} X_{A,\underline{a}}{}^{\underline{b}} (\Gamma^{\underline{a}}{}_{\underline{b}})_{\underline{\alpha}}{}^{\underline{\beta}}\,. \tag{2.6}$$

We can choose the worldvolume connection so that X_A takes a convenient form. The freedom in choosing the connection allows us to set $\nabla_A u_a{}^{\underline{b}} u^{-1}{}_{\underline{b}}{}^{b} = \nabla_A u_{a'}{}^{\underline{b}} u^{-1}{}_{\underline{b}}{}^{b'} = 0$. This implies that $\nabla_A u_\alpha{}^{\underline{\beta}} u^{-1}{}_{\underline{\beta}}{}^{\beta} = \nabla_A u_{\alpha'}{}^{\underline{\beta}} u^{-1}{}_{\underline{\beta}}{}^{\beta'} = 0$ since these quantities are simply related by gamma matrices. The non-zero quantities that remain are $X_{A,a}{}^{b'}$ and $X_{A,\alpha}{}^{\beta'} = \frac{1}{2} X_{A,a}{}^{b'} (\Gamma^a \Gamma_{b'})_\alpha{}^{\beta'}$. As we shall see below $X_{\alpha,a}{}^{b'}$ is related to the fermion field $\Lambda_a{}^{\alpha'}$ while $X_{a,b}{}^{c'}$ is the bosonic second fundamental form.

Analysing the torsion equation level by level in the linearised approximation we find the supervariations of h (dimension 1/2), Λ (dimension 1) which imply the supervariation of X. These are all given below in the two-step notation,

$$\nabla_{\alpha i} h_{abc} = -\frac{i}{8} \Lambda_{[a\beta i} (\gamma_{bc]})^{\beta}{}_{\alpha}\,, \tag{2.7}$$

$$\nabla_{\alpha i} \Lambda_{b\delta}{}^{l} = -\frac{1}{2} X_{b,a}{}^{c'} (\gamma^a)_{\alpha\delta} (\gamma_{c'})_i{}^l + \nabla_b h_{cde} (\gamma^{cde})_{\alpha\delta} \delta_i^l\,, \tag{2.8}$$

$$\nabla_{\alpha i} X_{b,a}{}^{c'} = -i(\gamma^{c'})_i{}^j \nabla_b \Lambda_{a\alpha j}\,. \tag{2.9}$$

At dimension $\frac{1}{2}$ we also have the relation of $X_{\alpha,a}{}^{b'}$ and $\Lambda_a{}^{\alpha'}$ which reads in two-step notation,

$$X_{\alpha i,a}{}^{b'} = i\Lambda_{a\alpha}{}^{j} (\gamma^{b'})_{ij}\,. \tag{2.10}$$

We also obtain the linearised equations of motion for the fermions (at dimension $\frac{1}{2}$) and for the scalars and tensor (at dimension 1),

$$(\gamma^a)^{\alpha\beta} \Lambda_{a\alpha i} = 0\,, \tag{2.11}$$

$$\eta^{ab} X_{a,b}{}^{c'} = 0\,, \tag{2.12}$$

$$\nabla^a h_{abc} = 0\,. \tag{2.13}$$

The last equation implies a Bianchi identity for h since h_{abc} is self-dual. In addition we find the constraints $X_{[a,b]}{}^{c'} = \nabla_{[a} \Lambda_{b]\delta}{}^{l} = 0$. These equations can be summarised by the statements that, in the linearised approximation, the fields h, Λ and X and their derivatives lie in the following representations of $\mathrm{Spin}(1,5) \times \mathrm{Spin}(5)$:

$$\nabla_{a_1 \ldots a_n} h_{bcd} \in (n02) \times (00)\,, \tag{2.14}$$

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$$\nabla_{a_1\ldots a_{n-1}}\Lambda_{a_n\alpha i} \in (n01)\times(01)\,, \tag{2.15}$$

$$\nabla_{a_1\ldots a_{n-2}}X_{a_{n-1},a_n}{}^{a'} \in (n00)\times(10)\,. \tag{2.16}$$

The irreducible representations are given in highest weight notation in the form $(abc)\times(de)$ with a,b,c Dynkin labels of Spin(1,5) (D3 in the Cartan classification) and d,e Dynkin labels of Spin(5) (B2 in the Cartan classification).

The full non-linear equations which follow from the embedding condition give the equations of motion, supervariations and components of the worldvolume torsion to all orders in number of fields as described in [2]. Here we require only the linearised analysis since we use a perturbative approach as in [9] to construct derivative deformations. Thus we regard the equations of motion as being determined order by order in terms of the fields of the linearised theory in the representations given above.

3. Derivative corrections

There is only one way to adapt the analysis of the previous section so that derivative corrections are included, namely to relax the embedding condition and allow $E_\alpha{}^{\underline{a}}$ to be a function of the fields. This approach was first used for the membrane in eleven dimensions in [8] and a similar one (involving deformations of the $\mathcal{F}$-constraint instead of the embedding condition) for the D9-brane of IIB in [9]. Deforming $E_\alpha{}^{\underline{a}}$ requires the introduction of an explicit length scale, l, since $E_\alpha{}^{\underline{a}}$ has dimension $-1/2$ while the fields h, Λ and X have dimensions 0, 1/2 and 1 respectively. As in [8] we parametrise the deformation by $\psi_\alpha{}^{a'}$ so that

$$E_\alpha{}^{\underline{a}} = \psi_\alpha{}^{a'}u_{a'}{}^{\underline{a}}\,. \tag{3.1}$$

The quantity ψ is only defined up to field redefinitions as discussed in [8] . The basic fields are the embedding coordinates $z^{\underline{M}}$ and the effect of the redefinition $z^{\underline{M}} \longrightarrow z^{\underline{M}} + (\delta z)^{\underline{M}}$ will define the ambiguity in ψ. The field redefinition is equivalent to a target space diffeomorphism and under such a transformation, given by a vector field v, the target space frame transforms as

$$\delta_v E^{\underline{A}} = (di_v + i_v d)E^{\underline{A}} = dv^{\underline{A}} + E^{\underline{C}}v^{\underline{B}}T_{\underline{BC}}{}^{\underline{A}}\,. \tag{3.2}$$

Applying the pullback map gives the transformation of the embedding matrix,

$$\delta_v E_A{}^{\underline{A}} = \nabla_A v^{\underline{A}} + E_A{}^{\underline{C}}v^{\underline{B}}T_{\underline{BC}}{}^{\underline{a}}\,. \tag{3.3}$$

Setting $A=\alpha$ and $\underline{A}=\underline{a}$ we find the transformation of ψ,

$$\delta_v\psi_\alpha{}^{c'} = \nabla_\alpha\hat{v}^{c'} + \hat{v}^b X_{\alpha,b}{}^{c'} - i\hat{v}^{\delta'}(\Gamma^{c'})_{\delta'\alpha} - ih_\alpha{}^{\epsilon'}\hat{v}^\delta(\Gamma^{c'})_{\delta\epsilon'}\,, \tag{3.4}$$

where $\hat{v}^{\underline{a}} = v^{\underline{b}}u_{\underline{b}}{}^{\underline{a}}$ and $\hat{v}^{\underline{\alpha}} = v^{\underline{\beta}}u_{\underline{\beta}}{}^{\underline{\alpha}}$.

Choosing $\hat{v}^a = \hat{v}^\alpha = 0$ gives

$$\delta_v\psi_\alpha{}^{c'} = \nabla_\alpha\hat{v}^{c'} - i\hat{v}^{\delta'}(\Gamma^{c'})_{\delta'\alpha}\,. \tag{3.5}$$

The second term allows us to remove the gamma-trace part of ψ so that we only need to look for deformations in the $(001)\times(11)$ representation of $\mathrm{Spin}(1,5)\times\mathrm{Spin}(5)$. The first term implies that those $\psi_\alpha{}^{c'}$ which are given by $\nabla_\alpha V^{c'}$ (projected onto the $(001)\times(11)$ representation) for some vector $V^{c'}$ are trivial deformations which can be removed by field redefinitions.

Hence we find the equivalence,

$$\psi_\alpha{}^{c'} \cong \psi_\alpha{}^{c'} + \nabla_\alpha V^{c'}\,. \tag{3.6}$$

As well as the field redefinitions which describe the above ambiguity in ψ there are constraints which ψ must satisfy. We find these by examining the dimension zero part of the torsion equation in the presence of non-zero ψ which reads

$$\nabla_{\alpha i}(\psi_{\beta j}{}^{c'}u_{c'}{}^{\underline{c}}) + (\beta j \leftrightarrow \alpha i) + T_{\alpha i\beta j}{}^{c}u_{c}{}^{\underline{c}} + T_{\alpha i\beta j}{}^{\gamma k}\psi_{\gamma k}{}^{d'}u_{d'}{}^{\underline{c}} = E_{\alpha i}{}^{\underline{\alpha}}E_{\beta j}{}^{\underline{\beta}}T_{\underline{\alpha}\underline{\beta}}{}^{\underline{c}}\,. \tag{3.7}$$

Applying $u_{\underline{c}}{}^{c}$ gives the dimension zero part of the worldvolume torsion. Applying $u_{\underline{c}}{}^{c'}$, we find the constraints which ψ must satisfy for the deformation to be consistent,

$$\nabla_{\alpha i}\psi_{\beta j}{}^{c'} + (\beta j \leftrightarrow \alpha i) = -i(\gamma^{c'})_{il}h_{\beta j\alpha}{}^{l} + (\beta j \leftrightarrow \alpha i)\,. \tag{3.8}$$

We split $h_{\alpha i\beta}{}^{j}$ into its irreducible representations,

$$\begin{aligned} h_{\alpha i\beta}{}^{j} \;=\;& \delta_i^j[h^a(\gamma_a)_{\alpha\beta} + h^{abc}(\gamma_{abc})_{\alpha\beta}] + (\gamma^{a'})_i{}^j[h^a_{a'}(\gamma_a)_{\alpha\beta} + h^{abc}_{a'}(\gamma_{abc})_{\alpha\beta}] + \\ &+(\gamma^{a'b'})_i{}^j[h^a_{a'b'}(\gamma_a)_{\alpha\beta} + h^{abc}_{a'b'}(\gamma_{abc})_{\alpha\beta}]\,. \end{aligned} \tag{3.9}$$

In the case $\psi = 0$ we then find that only h^{abc} is non-zero. For general ψ we find the constraints

$$Y_a^{a';c'} \;=\; (\gamma_a)^{\alpha\beta}(\gamma^{a'})^{ij}\nabla_{\alpha i}\psi_{\beta j}{}^{c'} = 0\,, \tag{3.10}$$
$$Z_{abc}^{a'b';c'} \;=\; (\gamma_{abc})^{\alpha\beta}(\gamma^{a'b'})^{ij}\nabla_{\alpha i}\psi_{\beta j}{}^{c'} = 0\,, \tag{3.11}$$

where in the first line it is to be understood that one keeps only the $(100)\times(20)$ representation of $\mathrm{Spin}(1,5)\times\mathrm{Spin}(5)$ and in the second line only the $(002)\times(12)$ representation.

At lowest order in the deformation and lowest order in the number of fields, the algebra of spinorial derivatives is the standard flat superspace algebra,

$$[\nabla_{\alpha i}, \nabla_{\beta j}] = i\eta_{ij}(\gamma^a)_{\alpha\beta}\nabla_a\,, \tag{3.12}$$

so that any $\psi_{\alpha i}{}^{c'}$ of the form $\nabla_{\alpha i}V^{c'}$ satisfies the constraints (3.10), (3.11) automatically up to higher orders. In the above formula η_{ij} is the antisymmetric $\mathrm{Spin}(5)\cong Usp(4)$ invariant antisymmetric tensor.

Thus we can consider the sequence of representations of $\mathrm{Spin}(1,5)\times\mathrm{Spin}(5)$,

$$(000)\times(10) \xrightarrow{\Delta} (001)\times(11) \xrightarrow{\Delta} ((100)\times(20))\oplus((002)\times(12))\,. \tag{3.13}$$

The operation Δ is given by applying a spinorial derivative and projecting onto the target representation. The algebra (3.12) implies Δ is nilpotent, $\Delta^2 = 0$. The analysis above can be summarised by saying that the genuine deformations of ψ away from zero are Δ-closed (they satisfy the constraints) and equivalent if they differ by Δ-exact terms (field redefinitions). Hence we are looking for ψ in the cohomology of Δ,

$$\psi \in H = \frac{\mathrm{Ker}\Delta}{\mathrm{Im}\Delta}\,. \tag{3.14}$$

We now explicitly calculate the cohomology H, working order by order in the number of fields. We must construct ψ in the representation $(001) \times (11)$ from the fields and their derivatives constrained by the linearised analysis (2.14), (2.15), (2.16). Also we must check possible field redefinitions in the representation $(000) \times (10)$ and constraints in the representations $(100) \times (20)$ (Y-constraints) and $(002) \times (12)$ (Z-constraints). It is obvious that there can be nothing for ψ linear in the fields since there is nothing in the correct representation. In fact this is already enough to rule out induced Einstein-Hilbert terms in the effective action.

At quadratic order in fields we find

$$\psi_{\alpha i}{}^{a'} = l^{2n-1}\nabla_{a_1}\dots\nabla_{a_{n-1}}\Lambda_{a_n\alpha i}\nabla^{a_1}\dots\nabla^{a_{n-2}}X^{a_{n-1},a_n a'}\,, \qquad \text{with } n \geq 2\,. \tag{3.15}$$

There are no field redefinitions $V^{a'}$ quadratic in the fields but there are constraints of the Y-type,

$$Y_a^{a';b'} = l^{2n-1}\nabla_a\nabla_{a_1}\dots\nabla_{a_{n-2}}X_{a_{n-1},a_n}{}^{a'}\nabla^{a_1}\dots\nabla^{a_{n-2}}X^{a_{n-1},a_n b'} \qquad \text{with } n \geq 2\,. \tag{3.16}$$

It is then simple to see that for each n applying a spinorial derivative and projecting onto the Y-representation gives a non-zero answer and hence none of the quadratic ψ are Δ-closed. Thus the cohomology quadratic in fields is trivial.

Moving on to cubic order in fields we find the first possibilities at order l^2. There are three linearly independent deformations, ψ, two possible field redefinitions, V, and two constraints, one each of Y-type and Z-type. The explicit formulae for these are given in appendix A. We find that two linearly independent combinations of the deformations can be removed by field redefinitions and the remaining combination gives a non-zero contribution to the constraints. Thus the cubic cohomology at order l^2 is trivial.

There is nothing one can write down for ψ at order l^3 which is cubic in fields and so the next order to check is l^4. Here we find 18 linearly independent deformations, ψ. There are 8 possible field redefinitions, V and 16 constraints, 8 each of Y-type and Z-type. The details of these are given in appendix B. At this order we find that 8 linearly independent combinations of the deformations can be removed by field redefinitions. Of the remaining 10 combinations only 2 are closed in the Y-sense. Finally, of these 2 only one is also closed in the Z-sense. It can be written,

$$\begin{aligned}\psi_{\alpha i}{}^{a'} = l^4[&4i\Lambda_{a\alpha}{}^j\nabla_b X_c{}^{ab'}X^{b,ca'}(\gamma_{b'})_{ij} + \nabla_a\nabla_b\Lambda_{c\alpha}{}^j\Lambda^a{}_\gamma{}^k\Lambda^b{}_\delta{}^l(\gamma^{a'b'})_{kl}(\gamma_{b'})_{ij}(\gamma^c)^{\gamma\delta} + \\ &+ 24i\Lambda_{a\beta i}X_{b,c}{}^{a'}\nabla^a\nabla^b h^{cde}(\gamma_{de})_\alpha{}^\beta]\,.\end{aligned} \tag{3.17}$$

The above formula is the first supersymmetric deformation of the embedding condition for the M5-brane in a flat eleven-dimensional background. The fact that the deformation is cubic in fields implies that the resulting corrections to the equations of motion are also cubic in fields. In particular this means, in terms of effective actions, that the pure curvature corrections will be quartic in the second fundamental form, X, and hence quadratic in the Riemann curvature, $R \sim X^2$. We will now show how the equations of motion of the deformed theory can be calculated from the above formula.

4. Derivative corrections to the equations of motion

One can derive the deformed equations of motion by analysing the torsion equation level by level in dimension in the presence of a non-zero ψ. As we have seen at dimension zero this implies constraints that ψ must satisfy. It also determines the remaining representations present in $h_{\alpha i\beta}{}^{j}$,

$$\begin{aligned}
h_a &= 0\,, &(4.1)\\
h_a^{a'} &= \frac{i}{16}\eta^{ij}(\gamma_a)^{\alpha\beta}\nabla_{\alpha i}\psi_{\beta j}{}^{a'}\,, &(4.2)\\
h_a^{a'b'} &= \frac{i}{32}(\gamma^{[a'})^{ij}(\gamma_a)^{\alpha\beta}\nabla_{\alpha i}\psi_{\beta j}{}^{b']}\,, &(4.3)\\
h_{abc}^{a'} &= \frac{i}{2.4^3.6}(\gamma^{a'b'})^{ij}(\gamma_{abc})^{\alpha\beta}\nabla_{\alpha i}\psi_{\beta j b'}\,, &(4.4)\\
h_{abc}^{a'b'} &= \frac{i}{2^2.4^2.6^2}\epsilon^{a'b'c'd'e'}(\gamma_{abc})^{\alpha\beta}(\gamma_{c'd'})^{ij}\nabla_{\alpha i}\psi_{\beta j e'}\,. &(4.5)
\end{aligned}$$

At dimension 1/2 we find from the ${}_{\alpha i b}{}^{\underline{c}}$ part of the torsion equation, contracted with $(u^{-1})_{\underline{c}}{}^{c'}$, that the relation between $X_{\alpha i,b}{}^{c'}$ and Λ is modified,

$$X_{\alpha i,b}{}^{c'} = i\Lambda_{b\alpha}{}^{k}(\gamma^{c'})_{ik} + \nabla_b\psi_{\alpha i}{}^{c'}\,. \tag{4.6}$$

Note that the first term also contains a correction term, the gamma trace of Λ, which is only zero at lowest order (the linearised fermionic equation of motion). We write

$$\Lambda_{a\alpha i} = (\gamma_a)_{\alpha\beta}\Pi_i^{\beta} + \hat{\Lambda}_{a\alpha i}\,, \tag{4.7}$$

where $\hat{\Lambda}_{a\alpha i}$ is gamma-traceless and Π is the correction to the fermionic equation of motion. Employing these relations we also find from the ${}_{\alpha i\beta j}{}^{\underline{\gamma}}$ part of the torsion equation,

$$\begin{aligned}
&\Big[-\frac{i}{2}(\gamma_b)_{\alpha\epsilon}\Pi^{\epsilon k}(\gamma^{c'})_{ik}(\gamma^b)_{\beta\delta}(\gamma_{c'})_j{}^l - \frac{1}{2}\nabla_b\psi_{\alpha i}{}^{c'}(\gamma^b)_{\beta\delta}(\gamma_{c'})_j{}^l + (\nabla_{\alpha i}h_{\beta j\delta}{}^l)^{(3,1)}\Big]+ \\
&\qquad\qquad +\alpha i \leftrightarrow \beta j - i(\gamma^c)_{\alpha\beta}\eta_{ij}(\gamma_c)_{\delta\epsilon}\Pi^{\epsilon l} = 0\,.
\end{aligned} \tag{4.8}$$

The superscript $(3,1)$ means the part which is cubic in fields and first order in the deformation parameter, which for the term we have calculated is l^4. Contracting with $(\gamma_d)^{\alpha\beta}\eta^{ij}$ we find

$$-(\nabla_\alpha{}^l h_{abc})^{(3,1)}(\gamma_d\gamma^{abc})^\alpha{}_\delta + (\gamma^{a'})^{il}(\gamma_d\gamma_a)^\alpha{}_\delta\nabla_{\alpha i}h_{a'}^{a} + (\gamma^{a'})^{il}(\gamma_d\gamma_{abc})^\alpha{}_\delta\nabla_{\alpha i}h_{a'}^{abc}+ \tag{4.9}$$

$$+(\gamma^{a'b'})^{il}(\gamma_d\gamma_a)^{\alpha}{}_{\delta}\nabla_{\alpha i}h^{a}_{a'b'} + (\gamma^{a'b'})^{il}(\gamma_d\gamma_{abc})^{\alpha}{}_{\delta}\nabla_{\alpha i}h^{abc}_{a'b'} + 18i(\gamma_d)_{\delta\epsilon}\Pi^{\epsilon l} = 0\,.$$

Taking the gamma-trace by contracting with $(\gamma^d)^{\delta\eta}$ gives

$$27i\Pi^{\eta l} = -(\gamma_a)^{\alpha\eta}(\gamma^{a'})^{il}\nabla_{\alpha i}h^{a}_{a'} - (\gamma^{a'b'})^{il}(\gamma_a)^{\alpha\eta}\nabla_{\alpha i}h^{a}_{a'b'}\,, \tag{4.10}$$

which is the correction to the fermionic equation of motion.

At dimension one we find from the ${}_{\alpha ib}{}^{\gamma}$ part of the torsion equation,

$$(\nabla_{\alpha i}\Lambda_{d\gamma}{}^{k})^{(3,1)} = (\nabla_d h_{\alpha i\gamma}{}^{k})^{(3,1)} + (X_{d,\alpha i\gamma}{}^{k})^{(3,1)}\,. \tag{4.11}$$

Contracting with $\delta^i_k(\gamma_{ab}\gamma^d)^{\alpha\gamma}$ gives the correction to the tensor field equation,

$$(\nabla^c h_{abc})^{(3,1)} = \frac{1}{32}\nabla_{\alpha i}\Pi^{\delta i}(\gamma_{ab})_{\delta}{}^{\alpha}\,. \tag{4.12}$$

Contracting with $(\gamma^{a'})_k{}^i(\gamma^d)^{\alpha\gamma}$ gives the correction to the scalar equation of motion,

$$(\eta^{bc}X_{b,c}{}^{a'})^{(3,1)} = \frac{3}{4}\nabla_{\alpha i}\Pi^{\alpha k}(\gamma^{a'})_k{}^i + 2\nabla_b h^{ba'}\,. \tag{4.13}$$

Thus all the corrections to the equations of motion are specified once ψ is known. As an example we give here the pure tensor field corrections to the tensor field equation,

$$\nabla^a h_{abc} = 120l^4(2\nabla_a\nabla_d\nabla_e h_{bcf}\nabla^a h_{gh}{}^{f}\nabla^d h^{egh} + 5\nabla_a h_{de}{}^{f}\nabla_g\nabla_h h_{bcf}\nabla^a\nabla^g h^{hde})\,. \tag{4.14}$$

We should stress that although we have given purely bosonic terms for simplicity, all corrections to the equations of motion are specified by this method, including all fermion terms.

5. Relation to deformations of $N = (2,0)$, $D = 6$ tensor gauge theory

There is an alternative way to derive deformations of the dynamics of the M5-brane by applying the spinorial cohomology techniques directly, without the superembedding [11]. In this section we sketch how the first deformation, corresponding to the leading contributions in the non-linear field equations (without derivative corrections) is derived this way. In principle our results of the previous sections could be derived from this approach, but we hope to illustrate with this short recap that the simpler and more powerful approach is to deform the superembedding, because we get the non-linear (non-derivative) corrections "for free" and, additionally, the superembedding approach we presented keeps κ-symmetry and reparametrisation invariance manifest.

To derive the deformations of the previous section one could directly deform the superspace constraints for the $N = (2,0)$, $D = 6$ tensor multiplet [1]. This multiplet can be described by a closed 3-form, H, on $N = (2,0)$, $D = 6$ superspace whose non-zero components are

$$H_{\alpha i\beta jc} = -i(\gamma_c)_{\alpha\beta}W_{ij}\,, \tag{5.1}$$
$$H_{\alpha ibc} = (\gamma_{bc})_{\alpha}{}^{\beta}\lambda_{\beta i}\,, \tag{5.2}$$

as well as H_{abc} which is self-dual. The fields are constrained to satisfy their equations of motion,

$$\partial_a \partial^a W_{ij} = (\gamma^a)^{\alpha\beta} \partial_a \lambda_{\beta k} = \partial^c H_{abc} = 0\,. \tag{5.3}$$

The constraint $H_{\alpha i\beta j\gamma k} = 0$ and the closure condition $dH = 0$ imply the fields of the tensor multiplet satisfy their equations of motion. The worldvolume theory of the M5-brane is described by a deformation of the constraints for the $N = (2,0)$ tensor multiplet. If one ignores derivative corrections then the full non-linear deformation of the constraint $H_{\alpha i\beta j\gamma k} = 0$ which defines the non-linear tensor gauge theory of the M5-brane must be implied by the embedding condition of the superembedding formalism since the constraint $E_\alpha{}^{\underline{a}} = 0$ defines the full non-linear theory [2]. The analogous relation between the $\mathcal{F}$-constraint of the superembedding formalism and the non-linear deformation of the $D = 10$ abelian Yang-Mills constraints which defines Born-Infeld theory was developed in [14]. There the relation is much more direct since the two constraints take a similar form; they are both constraints on a superspace 2-form. Here the constraints are on objects of different dimension and carrying different representations of Spin$(1,5) \times$ Spin(5), $\psi_{\alpha i}{}^{a'}$ which is of dimension $-\frac{1}{2}$ and $H_{\alpha i\beta j\gamma k}$ which (if we stick to the dimensions we have used throughout) is of dimension $-\frac{3}{2}$. This situation is similar to the different descriptions of eleven-dimensional supergravity where one can apply superspace constraints to the torsion 2-form or, in the 4-form formulation, to the superspace 4-form.

In terms of cohomology the deformation of the tensor multiplet constraints was studied in [11]. The relevant representations of Spin$(1,5) \times$ Spin(5) in $H_{\alpha i\beta j\gamma k}$ which remain after conventional constraints are $(003) \times (03)$ and $(101) \times (11)$. The relevant field redefinitions, which are redefinitions of the purely spinorial part of the 2-form potential lie in the representations $(002) \times (02)$ and $(100) \times (10)$. Finally the constraints which follow from the Bianchi identity lie in the representations $(004) \times (04)$, $(102) \times (12)$ and $(200) \times (20)$. We thus have the complex,

$$\begin{array}{ccccc}
(002)\times(02) & \longrightarrow & (003)\times(03) & \longrightarrow & (004)\times(04)\\
& \searrow & & \searrow & \\
(100)\times(10) & \longrightarrow & (101)\times(11) & \longrightarrow & (102)\times(12)\\
& & & \searrow & \\
& & & & (200)\times(20)\,.
\end{array} \tag{5.4}$$

The arrows represent operators given by applying a spinorial derivative and projecting onto the target representation. The non-trivial deformations of the constraints lie in the cohomology of the combined operation defined on the reducible sum of the irreps in each column.

The fields of the tensor multiplet consist of the scalars, W_{ij} in the $(000) \times (10)$ representation, fermions $\lambda_{\alpha i}$ in the $(001) \times (01)$ representation and the self-dual field strength H_{abc} in the $(002) \times (00)$ representation. In our conventions these have dimensions -1, $-1/2$ and 0 respectively. By dimensional analysis and inspection of the possible representations we can see that the leading deformation is cubic in fields and of the form $\lambda W H$ or $\lambda\lambda\lambda$. This induces terms cubic in the field strength H_{abc} (but with no derivatives) in the tensor field equation as well as accompanying scalar and fermion terms. Such a deformation

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corresponds to the leading non-derivative correction, the analogue of the leading Born-Infeld correction for the tensor gauge theory. In the manifestly kappa-symmetric approach described in the previous sections these corrections are completely taken into account by the embedding condition. The deformations of the embedding condition, which we have discussed correspond to deformations of the tensor multiplet constraints which include derivatives and an explicit dimensionful parameter, l. It is interesting to note that the kappa-symmetric formulation of the problem is much more tractable than the standard cohomology approach for deforming and solving the Bianchi identity for the tensor multiplet.

6. Conclusions

We have shown how spinorial cohomology can be used to derive the leading derivative and curvature corrections to the equations of motion for the tensor supermultiplet which describes the worldvolume dynamics of the M5-brane. The method used is manifestly supersymmetric and kappa-symmetric as these symmetries are implemented geometrically by using superspaces for the eleven-dimensional background and six-dimensional worldvolume.

The corrections we have calculated are fixed by the deformation (3.17) of the embedding condition of the superembedding formalism. The deformation was derived working first order by order in number of fields, where it was shown that there are no deformations at first or second orders, and then order by order in the scale l. The first cubic terms appear at order l^4 which is consistent with previous results on the membrane [8] and open string theories and D-branes [15 – 20, 9] and indeed one could perform a double dimensional reduction of the M5-brane results to obtain those for the D4-brane of IIA.

One can continue the calculation presented here to higher order in fields and higher orders in l. It is interesting to note that in the case of the membrane there are no gauge field degrees of freedom in the worldvolume multiplet and therefore there is no dimension zero field strength which could appear in the deformation. This means that there are a finite number of possible terms for the deformation at any given order in l. Thus it is possible to calculate the complete deformation at a given order in l although the calculation of [8] restricts to cubic terms to make the computation more tractable. In the case of the M5-brane and D-branes which contain a gauge field in their worldvolume multiplet this is no longer the case and one is in principle restricted to performing the analysis order by order in fields as well as order by order in l. However it may be the case that one need only consider a finite number of combinations involving the dimension zero field strength and deduce results for all orders in fields at a given order in l, just as one can for the undeformed superembedding.

The calculation here has been performed in a flat background but one can easily generalise to general on-shell backgrounds of eleven-dimensional supergravity. In particular one can address the issue of kinetic terms for the pullbacks of the background supergravity gauge fields in this approach while preserving all of the supersymmetry and kappa-symmetry. The corresponding terms for NS-NS and R-R fields for D-branes in type-II theories can also be derived either by dimensional reduction or directly using the same method.

Alternatively one can obtain correction terms by calculating string amplitudes directly as in [16, 21 − 25]. Such terms are relevant for the quantum polarisation discussed in [26].

A. Cubic cohomology at order l^2

Deformations. The deformations are given by $\psi_{\alpha i}{}^{a'}$ in the $(001)\times(11)$ irrep of $\mathrm{Spin}(1,5)\times\mathrm{Spin}(5)$. We have:

$$\psi_1 = \Lambda_{a\beta i} X^{a,ba'} h_{bcd} (\gamma^{cd})_\alpha{}^\beta\,, \tag{A.1}$$
$$\psi_2 = \Lambda_{a\beta i} X_{b,c}{}^{a'} h^{abd} (\gamma^c{}_d)_\alpha{}^\beta\,, \tag{A.2}$$
$$\psi_3 = \Lambda_{a\beta}{}^j \Lambda_{b\gamma}{}^k \Lambda_{c\delta i} (\gamma^c)^{\beta\gamma} (\gamma^{ab})_\alpha{}^\delta (\gamma^{a'})_{jk}\,. \tag{A.3}$$

Field redefinitions. The field redefinitions are given by $v^{a'}$ in the $(000)\times(10)$ irrep of $\mathrm{Spin}(1,5)\times\mathrm{Spin}(5)$,

$$V_1 = X_{a,b}{}^{a'} h^{acd} h^b{}_{cd}\,, \tag{A.4}$$
$$V_2 = \Lambda_{a\alpha}{}^i \Lambda_{b\beta}{}^j h^{abc} (\gamma_c)^{\alpha\beta} (\gamma^{a'})_{ij}\,. \tag{A.5}$$

Constraints. There are two irreps which contribute to the constraints. They are given by $Y_a{}^{a';b'}$ which is the $(100)\times(20)$ irrep of $\mathrm{Spin}(1,5)\times\mathrm{Spin}(5)$ and $Z_{abc}{}^{a';b'c'}$ which is the $(002)\times(12)$ irrep. At this order we have one of each:

$$Y_a{}^{a';b'} = \Lambda_{a\alpha i} \Lambda_{b\beta j} X^{b,ca'} (\gamma^{b'})^{ij} (\gamma_c)^{\alpha\beta}\,, \tag{A.6}$$
$$Z_{abc}{}^{a';b'c'} = \Lambda_{[a\alpha i} \Lambda_{|b|\beta j} X^b{}_c{}^{a'} (\gamma^{b'c'})^{ij} (\gamma_{d]})^{\alpha\beta} + \text{ dual }\,. \tag{A.7}$$

Spinorial relations. We apply a spinorial derivative followed by projection (which denote by Δ). We find:

$$\Delta V_1 = -\frac{i}{12}(\psi_1 + 2\psi_2)\,, \qquad \Delta V_2 = -2\psi_2 + \frac{i}{24}(\psi_3)\,, \tag{A.8}$$

and

$$\Delta\psi_1 = -iZ - \frac{i}{2}Y\,, \qquad \Delta\psi_2 = \frac{i}{2}Z + \frac{i}{4}Y\,, \qquad \Delta\psi_3 = 24Z + 12Y\,. \tag{A.9}$$

We see that we can use the two field redefinitions to eliminate two of the deformations and that the remaining one will have a non-zero contribution to the constraints and so is not closed. Therefore the cohomology at this order is trivial.

B. Cubic cohomology at l^4

Deformations. We have

$$\psi_1 = \Lambda_{a\alpha}{}^j \nabla_b X_c{}^{ab'} X^{b,ca'} (\gamma_{b'})_{ij}\,, \tag{B.1}$$
$$\psi_2 = \Lambda_{a\alpha}{}^j \nabla_b X_c{}^{aa'} X^{b,cb'} (\gamma_{b'})_{ij}\,, \tag{B.2}$$
$$\psi_3 = \Lambda_{a\beta}{}^j \nabla^a X_{b,c}{}^{b'} X^{b,da'} (\gamma_{b'})_{ij} (\gamma^c{}_d)_\alpha{}^\beta\,, \tag{B.3}$$
$$\psi_4 = \Lambda_{a\beta}{}^j \nabla^a X_{b,c}{}^{a'} X^{b,db'} (\gamma_{b'})_{ij} (\gamma^c{}_d)_\alpha{}^\beta\,, \tag{B.4}$$

$$\psi_5 = \nabla_a \Lambda_{b\alpha}{}^j X^{a,ca'} X^b{}_c{}^{b'} (\gamma_{b'})_{ij} \,, \tag{B.5}$$
$$\psi_6 = \nabla_a \Lambda_{b\beta}{}^j X^{a,ca'} X^{b,db'} (\gamma_{b'})_{ij} (\gamma_{cd})_\alpha{}^\beta \,, \tag{B.6}$$
$$\psi_7 = \Lambda_{a\beta i} \nabla^a X_{b,c}{}^{a'} \nabla^b h^{cde} (\gamma_{de})_\alpha{}^\beta \,, \tag{B.7}$$
$$\psi_8 = \Lambda_{a\beta i} \nabla_b X_{c,d}{}^{a'} \nabla^b h^{cae} (\gamma^d{}_e)_\alpha{}^\beta \,, \tag{B.8}$$
$$\psi_9 = \nabla_a \Lambda_{b\beta i} \nabla^a X^{b,ca'} h_{cde} (\gamma^{de})_\alpha{}^\beta \,, \tag{B.9}$$
$$\psi_{10} = \nabla_a \Lambda_{b\beta i} \nabla^a X_{c,d}{}^{a'} h^{bce} (\gamma^d{}_e)_\alpha{}^\beta \,, \tag{B.10}$$
$$\psi_{11} = \nabla_a \Lambda_{b\beta i} X^{a,ca'} \nabla^b h_{cde} (\gamma^{de})_\alpha{}^\beta \,, \tag{B.11}$$
$$\psi_{12} = \nabla_a \Lambda_{b\beta i} X_{c,d}{}^{a'} \nabla^a h^{bce} (\gamma^d{}_e)_\alpha{}^\beta \,, \tag{B.12}$$
$$\psi_{13} = \Lambda_{a\beta i} X_{b,c}{}^{a'} \nabla^a \nabla^b h^{cde} (\gamma_{de})_\alpha{}^\beta \,, \tag{B.13}$$
$$\psi_{14} = \nabla_a \nabla_b \Lambda_{c\alpha}{}^j \Lambda^a{}_\gamma{}^k \Lambda^b{}_\delta{}^l (\gamma^{a'b'})_{kl} (\gamma_{b'})_{ij} (\gamma^c)^{\gamma\delta} \,, \tag{B.14}$$
$$\psi_{15} = \nabla_a \Lambda_{b\beta}{}^j \nabla^a \Lambda^c{}_\gamma{}^k \Lambda_{d\delta i} (\gamma^{a'})_{jk} (\gamma^d)^{\beta\gamma} (\gamma^b{}_c)_\alpha{}^\delta \,, \tag{B.15}$$
$$\psi_{16} = \nabla_a \Lambda_{b\beta}{}^j \nabla^a \Lambda^c{}_\gamma{}^k \Lambda^b{}_{\delta i} (\gamma^{a'})_{jk} (\gamma^d)^{\beta\gamma} (\gamma_{cd})_\alpha{}^\delta \,, \tag{B.16}$$
$$\psi_{17} = \nabla_a \Lambda^j_{b\beta} \nabla^a \Lambda^b{}_\gamma{}^k \Lambda_{c\alpha}{}^l (\gamma^{a'b'})_{jk} (\gamma_{b'})_{il} (\gamma^c)^{\beta\gamma} \,, \tag{B.17}$$
$$\psi_{18} = \nabla_a \Lambda_{b\beta}{}^j \nabla^a \Lambda^c{}_\gamma{}^k \Lambda_{c\delta}{}^l (\gamma^{a'b'})_{jk} (\gamma_{b'})_{il} (\gamma_d)^{\beta\gamma} (\gamma^{bd})_\alpha{}^\delta \,. \tag{B.18}$$

Field Redefinitions

There are eight of these:

$$V_1 = X_{a,b}{}^{a'} X^{b,cb'} X_{c,}{}^a{}_{b'} \,, \tag{B.19}$$
$$V_2 = X_{a,b}{}^{a'} \nabla_c h^{ade} \nabla^c h^b{}_{de} \,, \tag{B.20}$$
$$V_3 = \nabla_a X_{b,c}{}^{a'} h^{ade} \nabla^b h^c{}_{de} \,, \tag{B.21}$$
$$V_4 = \Lambda_{a\alpha}{}^i \Lambda_{b\beta}{}^j \nabla^a X^{b,cb'} (\gamma^{a'}{}_{b'})_{ij} (\gamma_c)^{\alpha\beta} \,, \tag{B.22}$$
$$V_5 = \nabla_a \Lambda_{b\alpha}{}^i \Lambda^a{}_{\beta i} X^{b,ca'} (\gamma_c)^{\alpha\beta} \,, \tag{B.23}$$
$$V_6 = \nabla_a \Lambda_{b\alpha}{}^i \Lambda^a{}_\beta{}^j X^{b,cb'} (\gamma_c)^{\alpha\beta} (\gamma^{a'}{}_{b'})_{ij} \,, \tag{B.24}$$
$$V_7 = \nabla_a \Lambda_{b\alpha}{}^i \Lambda_{c\beta}{}^j \nabla^a h^{bcd} (\gamma_d)^{\alpha\beta} (\gamma^{a'})_{ij} \,, \tag{B.25}$$
$$V_8 = \nabla_a \Lambda_{b\beta}{}^j \nabla^a \Lambda_{c\gamma}{}^k h^{bcd} (\gamma_d)^{\beta\gamma} (\gamma^{a'})_{jk} \,. \tag{B.26}$$

Constraints. There are sixteen constraints, eight in each representation. For the representation $(100) \times (20)$ we have,

$$Y_1 = \nabla_b X_{c,d}{}^{a'} X^{b,eb'} \nabla^c h^d{}_{ea} \,, \tag{B.27}$$
$$Y_2 = \nabla_b \Lambda_{c\alpha i} \Lambda_{d\beta j} \nabla^b X^{c,da'} (\gamma^{b'})^{ij} (\gamma_a)^{\alpha\beta} \,, \tag{B.28}$$
$$Y_3 = \nabla_a \Lambda_{b\alpha i} \Lambda_{c\beta j} \nabla^b X^{c,da'} (\gamma^{b'})^{ij} (\gamma_d)^{\alpha\beta} \,, \tag{B.29}$$
$$Y_4 = \nabla_b \Lambda_{c\alpha i} \Lambda_{a\beta j} \nabla^b X^{c,da'} (\gamma^{b'})^{ij} (\gamma_d)^{\alpha\beta} \,, \tag{B.30}$$
$$Y_5 = \nabla_b \Lambda_{c\alpha i} \Lambda^b{}_{\beta j} \nabla^c X_{a,d}{}^{a'} (\gamma^{b'})^{ij} (\gamma^d)^{\alpha\beta} \,, \tag{B.31}$$
$$Y_6 = \nabla_b \nabla_c \Lambda_{d\alpha i} \Lambda^b{}_{\beta j} X^{c,da'} (\gamma^{b'})^{ij} (\gamma_a)^{\alpha\beta} \,, \tag{B.32}$$
$$Y_7 = \nabla_a \nabla_b \Lambda_{c\alpha i} \Lambda^b{}_{\beta j} X^{c,da'} (\gamma^{b'})^{ij} (\gamma_d)^{\alpha\beta} \,, \tag{B.33}$$
$$Y_8 = \nabla_a \Lambda_{b\alpha i} \nabla^b \Lambda^c{}_{\beta j} X_{c,d}{}^{a'} (\gamma^{b'})^{ij} (\gamma^d)^{\alpha\beta} \,. \tag{B.34}$$

For the representation $(002)\times(12)$ we have,

$$\begin{aligned}
Z_1 &= X_{[a,}{}^{da'}X_b{}^{eb'}\nabla_{c]}X_{d,e}{}^{c'} + \text{ dual}, && \text{(B.35)}\\
Z_2 &= \nabla_d\Lambda_{e\alpha i}\Lambda_{[a\beta j}\nabla_b X^{d,ec'}(\gamma^{a'b'})^{ij}(\gamma_{c]})^{\alpha\beta} + \text{ dual}, && \text{(B.36)}\\
Z_3 &= \nabla_d\Lambda_{[a\alpha i}\Lambda_{|e|\beta j}\nabla_b X^{d,ec'}(\gamma^{a'b'})^{ij}(\gamma_{c]})^{\alpha\beta} + \text{ dual}, && \text{(B.37)}\\
Z_4 &= \nabla_d\Lambda_{[a\alpha i}\Lambda_{b\beta j}\nabla_{c]}X^{d,ec'}(\gamma^{a'b'})^{ij}(\gamma_e)^{\alpha\beta} + \text{ dual}, && \text{(B.38)}\\
Z_5 &= \nabla_d\Lambda_{e\alpha i}\Lambda_{f\beta j}\nabla^d X^{e,fc'}(\gamma^{a'b'})^{ij}(\gamma_{abc})^{\alpha\beta} && \text{(B.39)}\\
Z_6 &= \nabla_d\nabla_e\Lambda_{[a\alpha i}\Lambda^d{}_{\beta j}X_{b,}{}^{ec'}(\gamma^{a'b'})^{ij}(\gamma_{c]})^{\alpha\beta} + \text{ dual}, && \text{(B.40)}\\
Z_7 &= \nabla_d\nabla_e\Lambda_{f\alpha i}\Lambda^d{}_{\beta j}X^{e,fc'}(\gamma^{a'b'})^{ij}(\gamma_{abc})^{\alpha\beta}, && \text{(B.41)}\\
Z_8 &= \nabla_d\Lambda_{e\alpha i}\nabla^d\Lambda_{[a\beta j}X^{e,}{}_b{}^{c'}(\gamma^{a'b'})^{ij}(\gamma_{c]})^{\alpha\beta} + \text{ dual}. && \text{(B.42)}
\end{aligned}$$

Spinorial relations. Applying a spinorial derivative and a projection to $V_1,\ldots,V_8$ we find

$$\begin{aligned}
\Delta V_1 &= 2i\psi_5\,, && \text{(B.43)}\\
\Delta V_2 &= \frac{i}{12}[\psi_{11}+2\psi_{12}]\,, && \text{(B.44)}\\
\Delta V_3 &= \frac{i}{24}[\psi_7-2\psi_8+\psi_9+2\psi_{10}]\,, && \text{(B.45)}\\
\Delta V_4 &= -2\psi_1+2\psi_3+\psi_2-\psi_4+12\psi_7+i\psi_{14}\,, && \text{(B.46)}\\
\Delta V_5 &= \frac{1}{2}\psi_1+\frac{1}{2}\psi_3+6\psi_{13}-\frac{1}{2}\psi_5+\frac{1}{2}\psi_6-6\psi_{11}\,, && \text{(B.47)}\\
\Delta V_6 &= -\psi_2-\psi_4+\frac{1}{2}\psi_1+\frac{1}{2}\psi_3+6\psi_{13}-\frac{1}{2}\psi_5-\frac{3}{2}\psi_6+6\psi_{11}-\frac{3i}{4}\psi_{16}-\frac{i}{4}\psi_{18}\,, && \text{(B.48)}\\
\Delta V_7 &= \psi_8-\psi_{12}-\frac{i}{48}\psi_{15}-\frac{i}{96}\psi_{16}-\frac{i}{48}\psi_{17}+\frac{i}{96}\psi_{18}\,, && \text{(B.49)}\\
\Delta V_8 &= -2\psi_{10}+\frac{i}{12}\psi_{16}+\frac{i}{24}\psi_{15}\,. && \text{(B.50)}
\end{aligned}$$

Applying a spinorial derivative to $\psi_1,\ldots,\psi_{18}$ and projecting onto the representation $(100)\times(20)$ (which we collectively denote by Δ^Y) we find

$$\begin{aligned}
\Delta^Y\psi_1 &= -3iY_6\,, & \Delta^Y\psi_2 &= -3iY_2\,,\\
\Delta^Y\psi_3 &= 192Y_1-6iY_7+3iY_6\,, & \Delta^Y\psi_4 &= 192Y_1+6iY_3-3iY_2\,,\\
\Delta^Y\psi_5 &= 0\,, & \Delta^Y\psi_6 &= 6iY_8\,,\\
\Delta^Y\psi_7 &= -16Y_1+\frac{i}{2}Y_3\,, & \Delta^Y\psi_8 &= -8Y_1+\frac{i}{12}Y_2+\frac{i}{12}Y_3-\frac{i}{6}Y_4+\frac{i}{6}Y_5\,,\\
\Delta^Y\psi_9 &= -\frac{i}{2}Y_4\,, & \Delta^Y\psi_{10} &= \frac{i}{6}Y_5+\frac{i}{12}Y_2-\frac{i}{6}Y_3+\frac{i}{12}Y_4\,,\\
\Delta^Y\psi_{11} &= 16Y_1+\frac{i}{2}Y_8\,, & \Delta^Y\psi_{12} &= -8Y_1-\frac{i}{4}Y_8\,,\\
\Delta^Y\psi_{13} &= \frac{i}{2}Y_7\,, & \Delta^Y\psi_{14} &= -12Y_6+12Y_7\,,\\
\Delta^Y\psi_{15} &= -4Y_2+4Y_4-8Y_5+16Y_8\,, & \Delta^Y\psi_{16} &= 4Y_2-4Y_3+8Y_5-8Y_8\,,\\
\Delta^Y\psi_{17} &= 12Y_2-12Y_4\,, & \Delta^Y\psi_{18} &= -24Y_5+12Y_2-12Y_3\,.
\end{aligned} \qquad \text{(B.51)}$$

One can easily check that these relations are consistent with each other in that $\Delta^Y\Delta$ must be identically zero. The set $\{\psi_3,\psi_4,\psi_5,\psi_8,\psi_{10},\psi_{11},\psi_{16},\psi_{17}\}$ can consistently be removed by field redefinition. Of the ten remaining ψ the following linear combinations satisfy closure in the Y-sense,

$$4i\psi_1 + 24i\psi_{13} + \psi_{14}\,, \tag{B.52}$$

$$8i\psi_2 + 6i\psi_6 + 24i\psi_7 - 24i\psi_9 - 48i\psi_{12} + 3\psi_{15} - \psi_{18}\,. \tag{B.53}$$

To check closure in the Z-sense we apply a spinorial derivative to these ψ followed by a projection onto the representation $(002) \times (12)$ (which we collectively denote by Δ^Z). We find

$$\Delta^Z\psi_1 = iZ_7\,, \qquad \Delta^Z\psi_{13} = -\frac{i}{6}Z_7 + iZ_6\,, \qquad \Delta^Z\psi_{14} = 24Z_6\,, \tag{B.54}$$

and so the first of the two linear combinations satisfies closure fully and is hence a non-trivial element of the cohomology H. The second combination fails Z-closure as one can see by the fact that the ψ_6 term is the only term which makes a non-vanishing contribution to the constraint Z_1.

We found the program LiE [27] useful to check that we have the correct number of terms in each representation.

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